

Assignment 4
Analytic Number Theory
MATH 773
Spring 2006
Due 3 Mar

1. (ch. 4, problem 25 in text) Show that the prime number theorem in the form $\psi(x) \sim x$ implies Selberg's asymptotic formula of theorem 4.18, but with error term replaced by $o(x \log x)$. I.e., show PNT implies

$$\lim_{x \rightarrow \infty} \left\{ \frac{\psi(x)}{x} + \frac{1}{x \log x} \sum_{n \leq x} \Lambda(n) \psi\left(\frac{x}{n}\right) \right\} = 2.$$

2. (ch. 6, part of problem 15 in text) Let χ be an odd character of conductor k . Show that for any integers $a < b$,

$$\left| \sum_{n=a}^b \chi(n) \right| \leq \frac{1}{2} \varphi(k)$$