**Assignment 2**
**Analytic Number Theory**
**MATH 773**
**Spring 2006**
**Due 3 Feb**

1. (ch. 2, problem 18 in text) Recall that a perfect number $n$ is one such that the sum of the proper divisors of $n$ is $n$, i.e., $\sigma_1(n) = 2n$. Show that if $2^a - 1$ is a prime, then $2^{a-1}(2^a - 1)$ is perfect.

2. (ch. 2, problem 14 in text) Let

$$f : [0,1] \cap \mathbb{Q} \to \mathbb{C}$$

be an arbitrary function. Set

$$F(n) = \sum_{k=1}^{n} f\left(\frac{k}{n}\right), \quad F^*(n) = \sum_{\substack{k=1 \\ (k,n)=1}}^{n} f\left(\frac{k}{n}\right).$$

   (a) Show the Dirichlet product $\mu * F = F^*$.

   (b) Use part (a) to show

$$\mu(n) = \sum_{\substack{k=1 \\ (k,n)=1}}^{n} e^{2\pi ik/n}.$$

3. (ch. 3, problem 2 in text) Prove that for $x \geq 2$,

$$\sum_{n \leq x} \frac{d(n)}{n} = \frac{1}{2}\log^2 x + 2\gamma \log x + O(1),$$

where $\gamma$ denotes Euler's constant.