

Assignment 1
Analytic Number Theory
MATH 773
Spring 2006
Due 20 Jan

1. (ch. 2, problem 4 in text) Show that for all $n \in \mathbb{Z}_+$ with no more than 8 prime factors,

$$\varphi(n) > \frac{n}{6}.$$

2. (ch. 2, problem 3 in text) Show for all $n \in \mathbb{Z}_+$,

$$\frac{n}{\varphi(n)} = \sum_{d|n} \frac{\mu(d)^2}{\varphi(d)}.$$

3. (ch. 2, problem 7 in text) Let $\mu(p, d)$ denote μ evaluated at the g.c.d. of p and d . Show that

$$\sum_{d|n} \mu(d)\mu(p, d) = \begin{cases} 1, & n = 1 \\ 2, & n = p^k, k \geq 1 \\ 0, & \text{otherwise} \end{cases}.$$