Direct-sum decompositions of modules over commutative rings

Suppose R is a commutative Noetherian ring. Let $\mathcal{C}$ be a class of finitely generated R-modules which is closed under finite direct sums and direct summands. For instance, $\mathcal{C}$ could be the class of torsion-free R-modules or the class of maximal Cohen-Macaulay R-modules. We talk about the following types of problems:

1. Suppose $X \in \mathcal{C}$. What is the structure of decompositions

   $$X = U_1 \oplus U_2 \oplus \cdots \oplus U_n$$

   of X into indecomposable modules $U_1, \ldots, U_n$? In particular, how non-unique are such decompositions?

2. How "large" can indecomposable modules $X \in \mathcal{C}$ be (in terms of invariants such as the multiplicity or the torsion-free rank)?

3. For which $X, Y, Z \in \mathcal{C}$ does the implication

   $$X \oplus Z \cong Y \oplus Z \Longrightarrow X \cong Y$$

   hold?

We sketch recent results for modules over one-dimensional rings obtained by A. Facchini, R. Karr, L. Klingler, R. Wiegand and myself.