A proportionally modular Diophantine inequality is an expression of the form $ax \mod b \leq cx$, with a, b y c positive integers such that $c < a < b$. The set $S = \mathrm{S}(a, b, c)$ of integer solutions to the above inequality is a numerical semigroup, that is, a subset of $\mathbf{N}$ closed under addition and with finite complement (in $\mathbf{N}$). This kind of numerical semigroups are called proportionally modular numerical semigroups.

We show how to solve this inequality and how to find a system of generators of this numerical semigroup studying its Bézout sequences. We also characterize those numerical semigroups and we study some of its properties as being irreducible, its number of gaps and we give a method to know when a numerical semigroup is a proportionally modular numerical semigroup. Finally, we present some open problems related to these inequalities.