

Mathematical Analysis Preliminary Examination Syllabus

- Measures
 - Definition and examples of σ -algebras.
 - Definition of Borel sets and the σ -algebra of Borel sets on a metric space.
 - Characterization of generators of the Borel sets in \mathbb{R} .
 - Hierarchy of F_σ and G_δ sets.
 - Definition of a measure and a measure spaces
 - Elementary properties of measures
 - Finite, σ -finite and semifinite measures
 - Complete measures
 - Premeasures on algebras, outer measures, and Caratheodory's theorem
 - Measurable sets
 - Characterizations of Lebesgue measurable sets
 - Product measures
 - Lebesgue measure
 - Properties of Lebesgue measure
 - Existence of non Lebesgue measurable sets
- Integration and Differentiation
 - Measurable functions
 - Characterizations of Borel measurable functions on \mathbb{R} .
 - Elementary properties of measurable functions
 - Simple functions
 - Approximation of measurable functions by simple functions
 - Definition of the integral with respect to a measure
 - Properties of the integral
 - Monotone convergence, Fatou's Lemma, and Dominated Convergence
 - Definition of integrable function
 - Properties of integrable functions
 - Approximation of integrable functions by simple functions and continuous functions
 - Relation between the Lebesgue and Riemann integral in \mathbb{R}
 - Convergence in measure
 - Egoroff's theorem and Lusin's theorem
 - Integration with respect to product measures
 - Fubini and Tonelli theorems
 - Signed and Complex measures
 - Absolute continuity and mutual singularity of measures
 - Radon Nikodym Theorem
 - Properties of the Radon Nikodym derivatives
 - Lebesgue Differentiation Theorem
 - Functions of bounded variation, and absolute continuity of functions
 - Fundamental theorem of calculus for Lebesgue integrals
- Topology
 - Metric spaces
 - Continuity of functions

- Tietze's Extension Theorem and Urysohn's Lemma
- Nets
- Compact spaces
- Tychonoff's theorem
- Arzela-Ascoli theorem
- Stone-Weierstrass theorem

- Functional Analysis
 - Normed vector spaces
 - Linear functionals
 - Hahn Banach theorem
 - L^p -spaces
 - Elementary properties of L^p -spaces
 - Hölder's and Minkowski's Inequality
 - Relationships between L^p spaces
 - Riesz representation theorem for L^p spaces