

Evaluation of Nonlinear Cumulative Damage Models for Assessing HCF/LCF Interactions in Multiaxial Loadings

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ABSTRACT

Past research has shown there is a nonlinear interaction effect between LCF (high damage) and HCF (low damage) cycles in many engineering materials. This effect has been observed within uniaxial loadings, but is often more pronounced under multiaxial loadings, particularly when the loading is non-proportional. This is an area of concern in the development of fatigue damage assessment methods for turbine engine materials, as typical mission spectra contain combinations of LCF and HCF cycles. The nonlinear interaction effect precludes the use of the most common technique for damage accumulation, the Palmgren-Miner linear damage rule.

A thorough review of nonlinear cumulative damage methodologies has been performed by the authors. These techniques have included simple extensions of the linear damage rule to include nonlinear terms, endurance-limit modification techniques, fracture-mechanics based approaches, and continuum damage approaches. While many methods have shown reasonable agreement with certain experimental results, none have previously been considered in conjunction with multiaxial data.

In this paper, a brief review of some well-known, nonlinear cumulative damage approaches is presented. The basic methodologies are discussed and evaluated in terms of accuracy with experimental results, range of applicability, and ease of implementation, with a particular focus on the extension of the theories to multiaxial loadings. Specific model evaluations are presented for a limited number of theories in comparison to experimental results generated from multiaxial mission-history tests and multi-block proportional load tests on smooth specimens of Ti-6Al-4V. Finally, obstacles that must be overcome in the development of fatigue damage assessment models for multiaxial mission loadings is discussed.

INTRODUCTION

In the development of a successful high cycle fatigue theory, there are two facets of fatigue damage that must be accounted for: (1) a method of correlating uniaxial and multiaxial data, and (2) a means of cyclic damage summation that accounts for load interaction effects, such as LCF-HCF interactions. Several definitions exist that attempt to meet the first of these requirements. Some of these models fall under the category of energy-based damage theories. They consider the increase in distortion energy density of a specimen as it progresses through its service life. After a critical number of cycles, this accumulated energy is sufficient to cause separation and failure of the component. While this may seem a logical approach, it neglects a crucial factor in the fatigue failure process. Since energy is a scalar quantity, the probability of failure along any interior plane is equal. However, empirical evidence has shown that specimens subjected to identical loadings typically fail in the same manner (i.e., along the same plane).

A second type of failure model manages to account for this phenomenon. Critical plane models calculate the amount of damage each interior plane experiences as a function of shear

and normal stresses or strains. The plane identified as possessing the greatest amount of damage is termed the critical plane. Since plane orientation is a directional quantity, these models describe along which plane a crack will initiate. It is this type of damage model, specifically the Findley model, that has been chosen by the authors in an attempt to generate a damage-life prediction curve for uniaxial data. A comparison of components subjected to multiaxial loadings can then be made against this curve. It should also be noted that attempts have been made to combine the aforementioned damage models into a third type of model that encompasses the best features of both. These energy-based critical-plane models have not been evaluated using the current data set and are only mentioned here for completeness.

Once the method of correlating uniaxial and multiaxial data has been determined, a means of calculating the amount of damage sustained per cycle must be established. Traditional methods of damage summation have been shown to provide an inaccurate life prediction when multiple load levels are simultaneously considered. This is due to the effect that one load level has on the other(s). In the present study, the effect of low-amplitude HCF loadings has had a more detrimental effect when coupled with the LCF loadings than predicted by a linear summation rule. Nonlinear damage accumulation theories can account for this influence and have shown an improvement in prediction. This paper includes data from two different sources to validate these theories. One set has been generated as part of the current HCF Program, and the other comes from testing performed by AEDC, Arnold AFB. The fatigue data were gathered using different multiaxial loading scenarios, and the comparisons are shown below.

FINDLEY MODEL

The method of evaluating the damage required for failure of a specimen is an important first step in the fatigue process. Previous work has detailed several damage theories and their application to both uniaxial and biaxial data [1]. The Findley model in particular has shown good correlation for biaxial Ti-6Al-4V data plotted against a uniaxial prediction curve. For ease, a summary of this model is provided here, although the reader is directed to [1] for a more detailed description.

The Findley parameter was a relatively early attempt to describe the amount of damage a material could sustain before failure. This model, among others, has the advantage of not only determining *when* a specimen will fail, but also *how* it will fail. It is what is termed a critical plane model. That is, failure can be determined by evaluating the damage caused by loading conditions on individual planes within the specimen or component. The plane with the greatest amount of damage is termed the critical plane and it is along this plane that the material should fail. Analytically, the Findley parameter is expressed as the sum of shear stress amplitude and the maximum normal stress (multiplied by a material constant). The general form of the Findley parameter is shown below.

$$FP = \frac{\Delta\tau}{2} + k\sigma_n^{\max} = AN^b + CN^d \quad (1)$$

where

- FP = Findley Parameter (damage)
- $\Delta\tau$ = shear stress range on the critical plane
- σ_n^{\max} = maximum normal stress on the critical plane

k = experimentally determined material constant to indicate the influence of the normal stress on the critical plane
 A, B, c, d = constants in the life prediction equation

The rightmost portion of Eq. (1) is a dual power-law relation detailing the damage-life prediction curve. In this study, the curve was generated using strictly uniaxial data to set a base value against which multiaxial data could be plotted. The shape of the curve was established by simultaneously optimizing the values of $k, A, b, C,$ and d to minimize the sum of the squared error between the experimental and predicted damage values. The intended result of this minimization procedure was the collapse of the data set around the prediction curve.

DATA SET 1: Ti-6Al-4V DATA FROM HCF PROGRAM

A large amount of uniaxial and biaxial (tension-torsion) fatigue data for Ti-6Al-4V was generated as part of the HCF Program. The uniaxial data consisted of $R = -1, 0.1,$ and 0.5 loadings, and the biaxial data included torsion, proportional, and non-proportional load paths. More details on the fatigue data can be found in [1]. The uniaxial fatigue data are shown in Fig. 1 in terms of the Findley parameter. The optimized terms for the prediction equation, using Eq. (1), are $k = 0.38, A = 52.9, b = -0.0186, C = 7191,$ and $d = -0.635$. The threshold line shown in Fig.1 was used as a lower bound for fatigue damage calculations; that is, values of $FP < 34$ ksi were assumed to cause no damage.

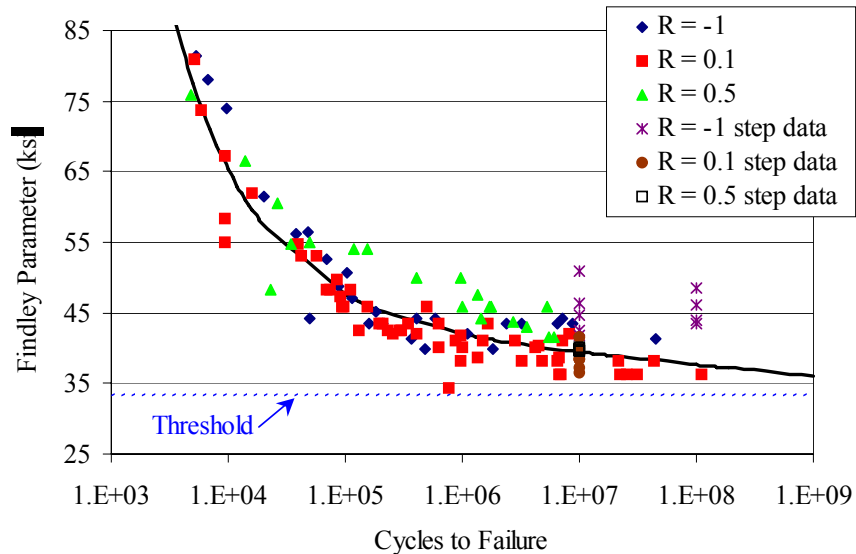


Figure 1. Uniaxial Ti-6Al-4V data from the HCF program, fit using the Findley model.

Once the curve-fit parameters were established for this data set, the damage and associated fatigue lives for specimens subjected to biaxial loadings were calculated and plotted against the uniaxial curve. While this presents little difficulty for proportional loading scenarios, non-proportional loadings are more complicated due to the varying definitions of the maximum normal stress, σ_n^{\max} , on the critical plane. In this study, two separate definitions were considered, which were found to significantly affect the accuracy of the Findley model in certain cases. In the first instance, σ_n^{\max} was defined as the largest value over the loading cycle. The

second definition considered σ_n^{\max} to take the maximum value of the normal stress at a shear stress reversal point. In the event that the maximum normal and shear stresses occur simultaneously, the two definitions are identical and correspond to the simpler loading scenario. Therefore, it is only when components are subjected to out-of-phase loadings that this distinction becomes important. Assuming crack initiation is a shear driven process, it is intuitive to utilize the normal stress associated with the maximum shear stress (i.e. at a shear reversal point). As shown in Ref. [1], this definition provided the best fit using the Findley model.

Figure 2 shows the biaxial data plotted against the uniaxial prediction curve. The plot indicates good agreement between experiment and prediction with two notable exceptions. First, the majority of the torsion data deviates somewhat from the prediction curve, erring on the side of non-conservative at longer lives. Second, the life of the 90° out-of-phase (circle path) test is noticeably over-predicted. Readers are referred to [1] for a more detailed discussion of these deviations. Despite these discrepancies, the overall capabilities of the Findley model in predicting biaxial fatigue lives is considered quite good.

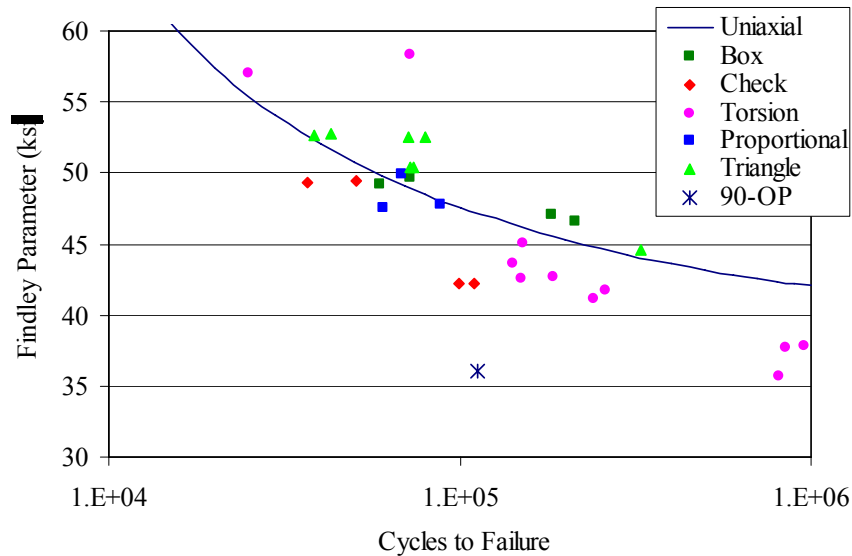


Figure 2. Plot of biaxial data from the HCF program against the uniaxial prediction curve.

Two biaxial load histories, or “mission histories”, were constructed to evaluate the interaction effects between high-damage (LCF) and low-damage (HCF) cycles for specimens of Ti-6Al-4V. These histories, termed the “check” path and “box” path, consist of an LCF cycle coupled with a number of prescribed HCF cycles and were designed to simulate the stresses that an aircraft engine component may experience during its service life. The two load paths are illustrated in Figure 3.

In both mission histories, an HCF subcycle was initiated at the point of maximum shear stress. The subcycles would then oscillate between this point and a predetermined lower value. Two test variants existed for box path testing: (1) one LCF cycle followed by 5 HCF subcycles, and (2) one LCF cycle followed by 50 HCF subcycles. The check mission was constructed from one LCF and 50 HCF cycles. Table 1 displays the average experimental LCF life and the predicted LCF and HCF lives for each condition. Predictions based on the linear damage rule indicate that the mission lives should be equal to the LCF lives for all cases. As can be seen

from the table, this holds true for the check path. That is, the HCF subcycles had negligible effect. However, the HCF cycles had a detrimental influence on the overall experimental life for the box path loadings. The reader is referred to [1] for a more detailed discussion.

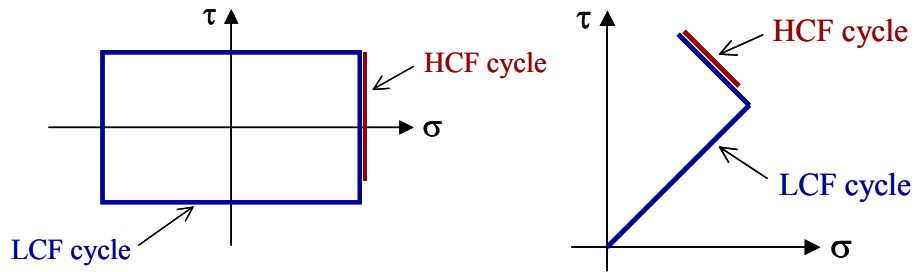


Figure 3. Depiction of the box path and check path mission histories.

Table 1. Experimental and Predicted LCF and HCF Lives for the Mission Histories

	Box 1	Box 2	Check
	1 LCF/ 50 HCF	1 LCF/ 5 HCF	1 LCF/ 50 HCF
Avg. Experimental LCF Life	65,900	65,900	43,700
Predicted LCF Life	66,100	66,100	66,800
Predicted HCF Life	2.5×10^8	2.5×10^8	2.5×10^8
Avg. Experimental Mission Life	20,400	44,100	47,200

The discrepancy of the high-cycle influence on the two mission histories indicates that damage is dependent on load path. Also, the difference between the LCF and mission fatigue lives for the box path seems to validate the theory that damage accumulates in a nonlinear manner. Therefore, an accurate life prediction model must account for these two phenomena. Below, a brief discussion for the correct assessment of damage related to the two forms of load paths is given. Later, methods to account for the nonlinear damage accumulation will be addressed.

Figures 4 and 5 show damage as a function of plane orientation for the box path and check path, respectively. The box mission possesses four maxima of approximately equal value for the LCF loadings. Based on this observation alone, the probability of failure along the planes of orientation associated with these maxima would be equal for the LCF cycle. However, a potential for interaction between the LCF and HCF cycles in Figure 4 can be observed, by noting that the peak damage from the HCF cycle occurs nearly coincidentally with one of the LCF peaks. Since the high-amplitude LCF cycles are the primary mechanism for failure, the contribution of the HCF cycles can be evaluated by determining the value of the HCF damage on the LCF plane. Figure 4 indicates that the coupled damage is greatest on the 12° plane, and it is along this orientation that failure should occur.

The check path depicts a decidedly different damage-orientation relation. While dissimilar in appearance, the same procedure for determining the plane of failure applies. This mission possesses two LCF and one HCF maxima. Again, examining the magnitude of the HCF cycle on the plane of greatest LCF damage shows that failure should occur on the 12° plane. However, in this instance the value of the HCF damage falls below the prescribed “threshold” value deemed

necessary to cause failure. Therefore, the effect of the HCF subcycles on the overall life of the component has been determined negligible for the check path.

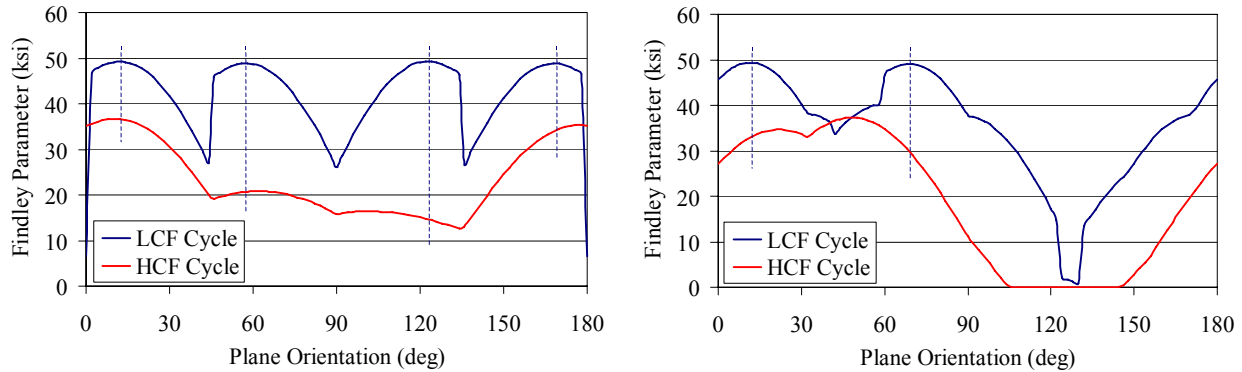


Figure 4 (left) and 5 (right). Damage as a function of plane orientation for the box mission (left) and check mission (right).

DATA SET 2: Ti-6Al-4V DATA FROM AEDC

The second set of data consisted of uniaxial fatigue data at stress ratios of $R = -1$ and 0.3 as well as a series of biaxial proportional loadings at ratios of -1 , 0.3 , and 0.5 . This data set was provided by Tom Tibbals of Sverdrup Technologies, AEDC, Arnold AFB. As with the previous set of data, a uniaxial prediction curve was first established to test the biaxial data against. Optimization of the Findley curve is depicted in Figure 6 and yielded the following values: $k = 0.464$, $A = 219650$, $b = -0.9286$, $C = 60.78$, and $d = 0$.

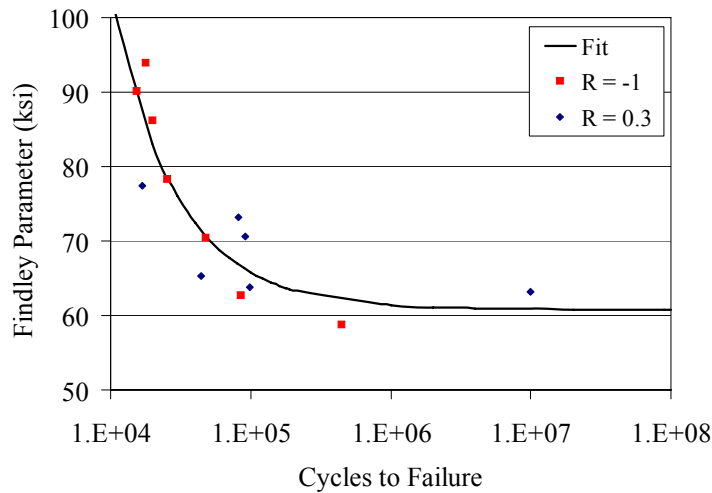


Figure 6. Uniaxial Ti-6Al-4V data from AEDC, fit using the Findley model.

The biaxial AEDC data are shown in Figure 7, along with the corresponding uniaxial curve fit from the Findley model. It is evident that the Findley model was less successful at correlating the biaxial data for this set than with the previous set of data. There also appears to be significant scatter within each load ratio for this set of data.

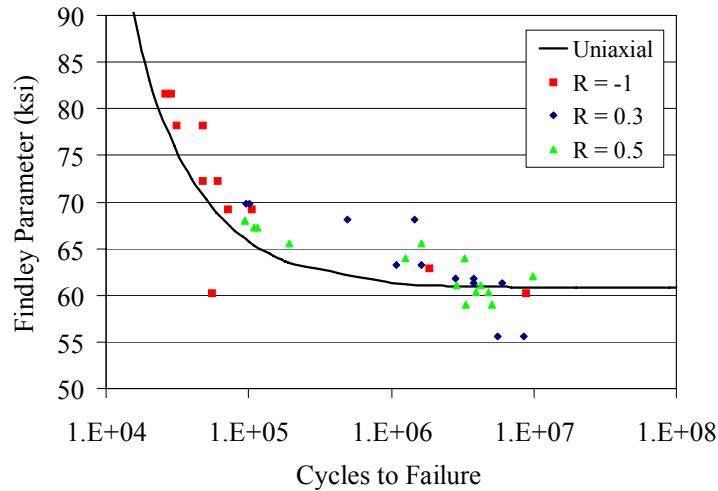


Figure 7. Biaxial AEDC data plotted against the uniaxial curve fit.

In addition to the single-level proportional tests, a series of biaxial, proportional multi-block loadings were also conducted on similar specimens of Ti-6Al-4V to investigate the nature of damage accumulation in this material. In these tests, the load ratio was held constant at $R = 0.3$, and the load magnitudes were incrementally increased or decreased until failure. These block-loading tests are illustrated schematically in Fig. 8.

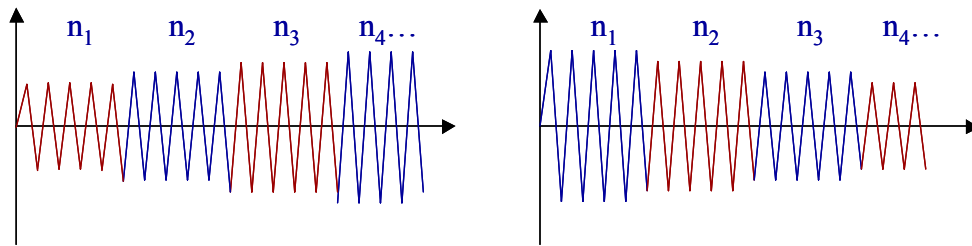


Figure 8. Depiction of increasing (low-high) and decreasing (high-low) block loadings.

The tests were designed to allow for eleven loading blocks. Each of the first ten blocks would be executed to the prescribed number of cycles. Then, the last loading block would be run out until failure. The stress levels were chosen to correspond to levels previously tested to failure, resulting in fatigue lives ranging from approximately 10^5 to 10^7 cycles. Consequently, the Findley model was not required in this case to predict fatigue lives corresponding to each stress level, removing the potential for additional uncertainties to be introduced during the damage summation calculations. Note that, since all load levels were similar in form, the critical plane would not vary from one cycle to the next.

Three different forms of these block loadings were conducted. Tests 1 and 2 consisted of 5,000 cycles per block, applied in an increasing fashion as depicted in the first example of Figure 7 (low-high). Test 3 was similar to tests 1 and 2, except that 15,000 cycles per block were applied. Finally, test 4 was conducted using the decreasing spectrum (high-low), as shown in the second example of Figure 7, and contained 15,000 cycles per block. A life summation based on the Linear Damage Rule (Miner's Rule) was performed on each of the loading histories. The results for the number of cycles to failure and the subsequent Miner's sum are listed in Table 2.

Table 2. Failure Lives and Miner's Sum for AEDC Block Loading Histories

Test	1	2	3	4
Consecutive Loadings (High - Low / Low - High)	L - H	L - H	L - H	H - L
Cycles / Block	5,000	5,000	15,000	15,000
Number of Blocks	11	11	11	6
Number of Cycles in Final Block	78,271	52,566	88,221	13,105
Miner's Sum	0.81	0.55	0.94	0.19

Examination of these results shows that test 4 failed at 13,105 cycles into block 6. This implies that most of the damage done in this test was from the initial high-amplitude loadings. In contrast, the first few loading blocks of tests 1 through 3 were relatively ineffectual in causing failure. The longer life of these increasing load tests may be due to having the stress levels associated with the first few loading blocks result in damage less than the threshold damage for this data set. Also note that the Miner's sum is less than 1 for all of the tests. Since this life summation is based on a linear rule, this is evidence that a nonlinear damage summation is required to properly define the fatigue process.

NONLINEAR CUMULATIVE DAMAGE MODELS

It has been shown that a linear summation of damage is often not adequate to predict the service life of a component when subjected to variable-amplitude loadings. In an effort to rectify this shortcoming, and thereby account for the second requisite for a successful life prediction model, the application of nonlinear cumulative damage models has been reviewed. Several nonlinear methods exist, including fracture mechanics, continuum-damage, and life-curve approaches. Each of these methods can be further subdivided into numerous theories having both advantages and drawbacks, depending on the scenario considered.

Some of the considerations that should be realized before a nonlinear model is chosen are ease of implementation, the independence of the stress state (i.e., multiaxial vs. uniaxial), and the independence of the fatigue damage parameter (e.g., Findley model). The authors have chosen to pursue life-curve approaches due to their compatibility with the aforementioned criteria, particularly ease of implementation. The specific theories evaluated are the Double Linear Damage Rule (DLDR), Damage Curve Approach (DCA), and Double Damage Curve Approach (DDCA). A brief discussion of these theories is presented here. Also included is a description of the Linear Damage Rule so a comparison may be made against this baseline model.

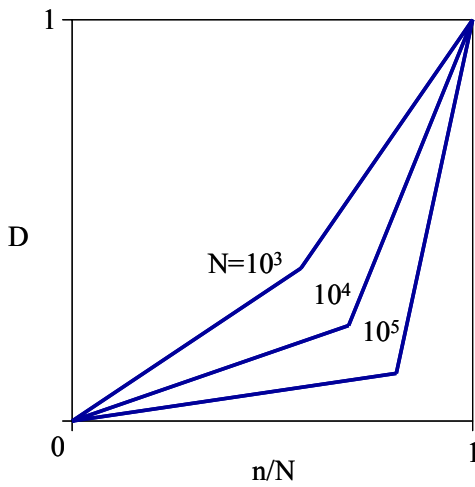
Linear Damage Rule

The most common method of summing damage for a loading spectrum is the Palmgren-Miner linear damage rule [2]. It is readily understood and easy to implement and is, therefore, the foundation for many of the other cumulative damage theories that have been proposed. A thorough discussion of the Palmgren-Miner rule is not needed, but it should be pointed out that, ideally, the summation of life ratios would equal one at failure. However, past experiments have yielded a range of ratios from 0.7 to 2.2 for uniaxial loadings, resulting in failure predictions erring just slightly on the side of nonconservative to more than double for a conservative prediction [3]. For the biaxial loadings presented here, a Miner's summation of 0.19 was found in one case, indicating extremely nonconservative results.

The largest drawback to the linear damage rule is its inability to account for the order of loading. That is, the resulting failure prediction is independent of the load interaction effects that have been observed between high-cycle and low-cycle loadings. It is this shortcoming that has prompted the development of several nonlinear cumulative damage theories.

Double Linear Damage Rule

The current form of the Double Linear Damage Rule, or DLDR, was proposed by S.S. Manson in 1966 [4]. Instead of a single straight line, a set of two straight lines that converged at a common “kneepoint” would be used. It was believed that this would help differentiate between the damage caused by the low-cycle and high-cycle regimes for multi-level loadings. The basis for the DLDR actually stems from an evaluation of damage curves; two straight lines replace a continuous curve. Each leg, or damage line, represents a separate stage in the failure process, Phase I (lower leg) and Phase II (upper leg). Individual two-level loadings can be plotted on the same damage versus applied cycles figure. When the loadings are normalized with respect to their failure lives a single equivalent plot is attained, as shown in Figure 9. This allows each phase of the loading to then be analyzed using the Palmgren-Miner linear damage rule.



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The difficulty encountered when utilizing the DLDR is establishing the location of the kneepoint, or transitory point between Phase I and Phase II loadings. A series of block loading tests must be performed for this determination. Manson and Halford [5] tested round steel specimens to experimentally assess the coefficients that would describe this point for a parameter based on the ratio of failure lives, N_1/N_2 . The resulting equations for the kneepoint are shown below in Eq. (2).

Figure 9. Double Linear Damage Rule.

$$\left. \frac{n_1}{N_1} \right]_{knee} = 0.35 \left[\frac{N_1}{N_2} \right]^\alpha \quad \left. \frac{n_2}{N_2} \right]_{knee} = 0.65 \left[\frac{N_1}{N_2} \right]^\alpha \quad (2)$$

It is also necessary to experimentally assess the value of the material parameter, α . Note that this value may not be the same as that used for other models. Every material needs to have its baseline parameters established for each model as their influence varies.

The versatility of the DLDR allows it to be extended to more than just two-level loading histories. Any number of load blocks may be included for analysis as long as one is careful how the common kneepoint is defined. A method for doing just this has been defined by Manson and Halford and yields results that are typically within 10 percent of either the damage curve approach or double damage curve approach without the added complexities of those models. It is also important to take into consideration the load sequence and load level when implementing this model. The DLDR yields different sets of prediction lines based on these parameters.

Damage Curve Approach

One of the first attempts to better describe fatigue failure using nonlinear damage was put forth by Richart and Newmark in 1948 [6]. Instead of a straight line, it was believed a single continuous curve would more accurately reflect the influence of the loading. This model, therefore, became known as the Damage Curve Approach or DCA. For low-amplitude, high-cycle loadings it was believed that a significant number of cycles had to be applied before enough damage could accumulate to cause a reduction in life. Once the appropriate number of cycles had been applied the damage continued to accumulate at an ever-increasing rate and failure was soon to follow. For high-amplitude, low-cycle loadings, this behavior was less pronounced. However, this early attempt did not have a constitutive equation to predict the relations between high- and low-cycle loadings, and it became apparent that a single curve was incapable of predicting both failure methods. After several revisions, Manson and Halford [5] were able to provide a workable equation based on early crack growth theories. The left side of Eq. (3) shows the damage function in its most general form. With some manipulation, the final form describing the damage curve in terms of a reference life, N_{ref} , was derived as shown on the right side of Eq. (3).

$$D = \left[\frac{1}{0.18} \right] \left[a_0 + (0.18 - a_0) \left(\frac{n}{N} \right)^{\left(\frac{2}{3} \right) (N)^\alpha} \right] \quad \text{or} \quad D = \left(\frac{n}{N_f} \right)^{\left(\frac{N_f}{N_{ref}} \right)^\alpha} \quad (3)$$

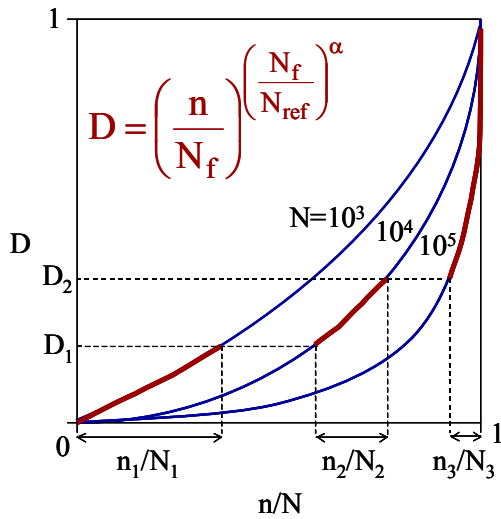


Figure 10. Damage Curve Approach.

Double Damage Curve Approach

Although the DCA shows tremendous potential in accurately predicting failure in multi-level loadings, there is one serious drawback when considering high-low loadings. Upon examination it can be seen that with the application of just a few high-amplitude cycles, there is a

rapid decrease in remaining life at the low-amplitude load level. This result is from a lack of the low-range data needed to adjust the shape of the curve during the models conception.

To improve the model, Manson and Halford [7] included a linear term to shift the curves away from the x-axis. The difficulty would be to allow this new term to have a significant influence at low life ratios but negligible effect at higher ratios. The resulting double damage curve approach (DDCA) closely approximated the DLDR in the lower-life regime and the DCA in the higher-life regime, where each model performed best. The equation for the DDCA is shown below.

$$D = \left(\frac{n}{N}\right) \left\{ q_1^\gamma + \left[1 - q_1^\gamma \left[\frac{n}{N} \right]^{\gamma(q_2-1)} \right]^\frac{1}{\gamma} \right\} \quad (4)$$

where

D = damage accumulated

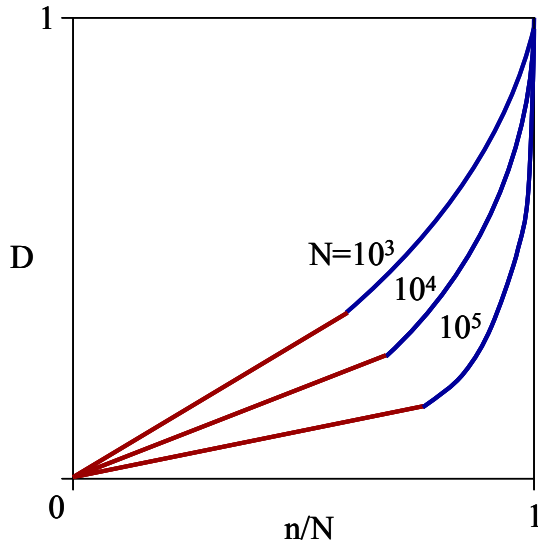
n = number of applied cycles at a given load level

N = number of cycles required to fail at the same load level as n

$$q_1 = \frac{0.35 \left(\frac{N_{ref}}{N}\right)^\alpha}{1 - 0.65 \left(\frac{N_{ref}}{N}\right)^\alpha} \quad \text{and} \quad q_2 = \left(\frac{N}{N_{ref}}\right)^\beta \quad (5)$$

$\gamma = 5$; a constant representing two intersecting straight lines which can be replaced by a single curve.

α, β = material dependent parameters that must be experimentally determined (typically taken as 0.25 and 0.4, respectively)



It is recognized that q_1 in this equation is the same result as the slope for the first straight line segment of the DLDR. Therefore, the coefficient values, 0.35 and 0.65, should be experimentally verified for each material and loading history and only taken at face value when such testing is precluded. It is also noted that q_2 is the exponential term that appears in the DCA model. Again, because of the changing influence of the material parameter, α (or β in this case) should be experimentally evaluated. The DDCA model is illustrated schematically in Fig. 11. Notice the linear damage accumulation at lower life ratios and curvilinear damage accumulation at higher life ratios.

Figure 11. Double Damage Curve Approach.

RESULTS AND COMPARISONS

In this study, an attempt was made to evaluate the three models described previously in terms of their ability to predict the fatigue lives of the mission histories and multi-block load histories. Due to the limited amount of data, the required material parameters for each model had to be determined from the variable load histories themselves. Thus, the analysis here focused on determining whether a single set of parameters could be found for each material that would result in good agreement for all the load cases considered. The results from this analysis are shown in Tables 3 (HCF Program data) and 4 (AEDC data) and discussed below. For comparison purposes, life predictions using the Linear Damage Rule (LDR) are also included.

The first damage theory examined was the Double Linear Damage Rule. Known values of n_1 , N_1 , n_2 , and N_2 from the HCF Program data were inserted into the kneepoint equations, Eq. 2, to establish a value for α . A solution for summing the damage from the constituent loading levels yielded an average α of approximately 0.0935. Note that this result is based on the kneepoint coefficients having values of 0.35 and 0.65, since there were insufficient test data for Ti-6Al-4V to establish more appropriate values. Failure lives were predicted using the Findley model and input into the DLDR. The resulting mission life predictions for the HCF Program data using the DLDR are shown in Table 3 below.

As is evident from Table 3, the DLDR is able to predict a reduction in the box mission lives due to the HCF cycles, although the predictions are nearly the same for the two box missions. This would imply that the first few HCF cycles cause significant damage, but their influence decreases as more cycles are applied. This is in contrast to the experimental results. Considering the check path, the model predicts no adverse effect from the HCF cycles. This result holds true for each nonlinear damage model discussed because, as mentioned earlier, the HCF damage on the LCF critical plane associated with this mission path falls below a predetermined value deemed necessary to be detrimental to the mission life.

An attempt was made to apply the DLDR model to the second data set (AEDC data). However, this proved to be very difficult, as the model requires two-level loadings to determine the kneepoint locations. The only variable load-level data available from this data set were multi-block loadings. Therefore, a reliable value of α could not be determined for this data set.

The next model to be evaluated was the Damage Curve Approach. This model has shown potential in adequately describing the effect of loadings in the high-cycle spectrum. Just as with the DLDR, the initial step for employing the DCA model was to ascertain the material dependant parameter, α . For this model, α was iteratively determined for each load history by summing the damage in Eq. (3) until the predicted and experimental fatigue lives converged. An average value for α was determined to be 0.79 for the HCF Program data. The AEDC data yielded a value of 1.28. However, the value for the second data set is not an average, as it was obtained strictly from test 4; tests 1 through 3 resulted in values of $\alpha < 0$. While theoretically possible, this would result in damage curves mirrored about the diagonal in Fig. 10.

From Tables 3 and 4, it is evident that the DCA model was capable of accounting for the nonlinear damage accumulation rates observed experimentally. The predictions for the two box mission lives (Table 3) are in excellent agreement with experimental results using the DCA model with $\alpha = 0.79$. Considering the AEDC data, when the value of $\alpha = 1.28$ (from test 4) was used in the life predictions for tests 1 – 3, there was very little deviation from the LDR calculations. While not in perfect agreement, the predicted lives for tests 1 and 2 are within reasonable bounds, and the prediction for test 3 is quite good (indicating the damage

accumulation rates for these tests can be reasonably approximated as linear). The prediction for test 4 is, as expected, in close agreement. Thus, it can be concluded that the DCA model, with a single value of α determined for each material, can accurately model the nonlinear damage accumulation in Ti-6Al-4V

The last model explored was the Double Damage Curve Approach. This cumulative damage theory took the best aspects of both the DLDR and DCA theories and combined them into one encompassing equation; thus, it would be expected that this model would give the best overall prediction. However, there is considerable difficulty when using this model in the high-cycle regime. The problem arises when one tries to determine the material parameter, α , or the predicted mission life. Following the same procedure to determine α that was employed for the DCA theory is not possible in this case due to the influence of the β and γ parameters. The LCF and HCF failure lives are taken to the β power and then multiplied by the geometric constant, γ . Neither data set contains the information necessary to solve these parameters uniquely. As a result, these multivariable equations can provide accurate predictions when at least one of the values is set. However, since that was not the case in the present study the results from this model were unreliable due to the unstable nature of the equation.

Table 3. Experimental and Predicted HCF Program Mission Lives using the Nonlinear Cumulative Damage Theories

	Box 1	Box 2	Check
	1 LCF/ 50 HCF	1 LCF/ 5 HCF	1 LCF/ 50 HCF
Experimental Results			
Average LCF Life	65,900	65,900	43,700
Average Mission Life	20,400	44,100	47,200
Predicted Mission Lives			
LDR	65,900	65,900	43,700
DLDR ($\alpha = 0.0935$)	29,658	30,694	43,700
DCA ($\alpha = 0.79$)	17,407	46,965	43,700
DDCA	--	--	--

Table 4. Experimental and Predicted AEDC Mission Lives using the Nonlinear Cumulative Damage Theories

	Tests 1 & 2	Test 3	Test 4
	L - H, 5,000 / Block	L - H, 15,000 / Block	H - L, 15,000 / Block
Experimental Results			
Number of Blocks	11	11	6
Cycles in Final Block	65,419	88,221	13,105
Predicted Lives (Final Block)			
LDR	97,076	94,131	4 x 10 ⁶
DLDR	--	--	--
DCA ($\alpha = 1.28$)	98,549	98,549	12,655
DDCA	--	--	--

SUMMARY AND CONCLUSIONS

A successful model for evaluating fatigue damage accumulation should be capable of accounting for different stress states (i.e., uniaxial vs. multiaxial), different load paths (e.g., proportional vs. nonproportional), and load sequence/load interaction effects (e.g., LCF/HCF interactions). The experimental data considered in this study show damage may accumulate nonlinearly under multiaxial loadings, but the accumulation rates depend on load path and load sequence. Although several possibilities may exist for a good fatigue model, the results here show that the coupling of a critical plane damage parameter (Findley) with a nonlinear damage accumulation model (DCA) yields a method capable of accounting for these factors.

The nonlinear models described herein were developed using two-level loadings. However, that does not preclude their use with multilevel loadings. To more fully understand their applicability, additional multiaxial fatigue data are required. Uniaxial data are needed to establish the model constants and variable load data would be used to fully evaluate their capabilities. As part of this latter data set, variable load histories comprised of more than two load levels (or cycles) should be included, as many cumulative damage models can be “fit,” through the proper selection of parameters, to two-level loadings. In other words, a truly discriminating evaluation of nonlinear cumulative damage models can only be undertaken when three or more points on the damage curve are considered. Under conditions of multiaxial fatigue, the different cycles could be obtained by varying the applied stress levels, or simply by varying the stress path between cycles. Ideally, both parameters would be varied.

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