

Multiaxial Fatigue Evaluation of Ti-6Al-4V under Simulated Mission Histories

Mr. Eric Goodin¹, Dr. Alan Kallmeyer¹, and Dr. Peter Kurath²

¹ Department of Mechanical Engineering, North Dakota State University, Fargo, ND

² Department of Mechanical Engineering, University of Illinois, Urbana, IL

ABSTRACT

The authors have previously evaluated over twenty fatigue damage parameters for application to titanium alloys (Ti-6Al-4V) subjected to constant amplitude loadings. Equivalent stress based parameters and critical plane parameters were considered for both proportional and non-proportional multiaxial load paths. The evaluations were based on the ability of the models to correlate tension/torsion fatigue data to a uniaxial baseline. Only a few models displayed good agreement for the range of experimental data. In the present study, the extension of the multiaxial fatigue parameters to simple variable load histories is considered. Several simulated mission histories were applied to solid round bars of Ti-6Al-4V under tension/torsion loading. The mission histories consisted of a single highly damaging LCF cycle (often non-proportional), coupled with 5 to 50 small, low-damage HCF cycles. Each specimen was subjected to an individual mission history applied repeatedly until failure. The differences between the HCF and LCF cycles incorporated both cycle “shape” (or path) and amplitude of loading. Comparisons between experimental and predicted fatigue lives will be presented. Specific attention will be given to the interaction effects between the HCF and LCF cycles. Also, the degree to which the load path influences the multiaxial fatigue damage will be discussed.

INTRODUCTION

A useful multiaxial fatigue model must be capable of accurately estimating the fatigue damage sustained from a wide variety of loading conditions. Many components in service are routinely subjected to complex cyclic multiaxial stresses, which may vary in time in a somewhat random manner. Due to the difficulty and cost of experimentally replicating such complex load histories in a controlled environment, the majority of multiaxial fatigue parameters currently in service have only been validated for relatively simple biaxial load paths in which the cyclic levels are held constant throughout the duration of the test. To place greater reliability in these models, further experimental validations are required that more closely simulate the actual load histories anticipated in service.

The consideration of variable amplitude load histories gives rise to some unique challenges in multiaxial fatigue life prediction, particularly concerning issues such as cycle counting and damage accumulation. Many multiaxial damage parameters include stress-strain terms from more than one tensorial location. The choice of what term in the damage parameter to use as the “cycle-counter” for the extension of an algorithm such as rainflow counting is not always obvious. The ambiguity is most evident when considering critical plane models, which typically define damage based on a particular combination of shear and normal stress or strain terms on a given plane. Furthermore, the question of damage accumulation becomes important when the “critical” plane (plane on which the highest level of damage occurs) for each cycle may differ.

COMPARISON OF MODELS USING CONSTANT AMPLITUDE DATA

In previous studies [1, 2], the authors have evaluated over 20 existing fatigue damage parameters by comparison to numerous Ti-6Al-4V uniaxial data and approximately thirty biaxial smooth bar tests which encompassed a variety of mean stress conditions and non-proportionality. Both the level of loading and shape of the paths considered in those studies were intended to simplify portions of actual service events. Figure 1 highlights some of the non-proportional load paths considered in the previous investigations. The reader is referred to reference [1] for other details of the testing and specimen design.

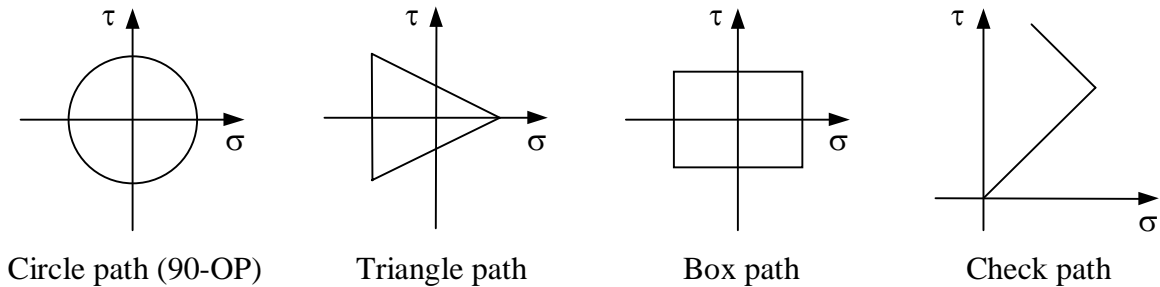


Figure 1. Non-proportional biaxial load paths.

Based on the results of the earlier studies, two multiaxial parameters were identified for further analysis. These two parameters were selected because they provided the best overall correlation of both the uniaxial and biaxial Ti-6Al-4V data considered in this program. The first parameter, the modified Manson-McKnight (MMM) model [3, 4], is an “effective-stress” type of parameter that uses an alternating stress range, $\Delta\sigma_{\text{eff}}$, defined from the general form of effective stress with the range of each stress component, and a mean stress term, $\sigma_{\text{eff}}^{\text{mean}}$, defined as the effective mean stress obtained using the mean stress for each component.

$$\text{Damage} = \frac{1}{2} \Delta\sigma_{\text{eff}}^{(w-1)} \sigma_{\text{max}}^w, \quad \sigma_{\text{max}} = \frac{\Delta\sigma_{\text{eff}}}{2} + \frac{\beta}{2} \sigma_{\text{eff}}^{\text{mean}}, \quad \beta = \frac{(\sum\sigma_1 + \sum\sigma_3)}{(\sum\sigma_1 - \sum\sigma_3)} \quad (1)$$

The sign and magnitude of the mean stress term in the model is modified by multiplying the effective mean by the term $\beta/2$. β is defined by $\sum\sigma_1$ and $\sum\sigma_3$, which are the sums of the first and third principal stresses, respectively, at the maximum and minimum points in the cycle. If strains (rather than stresses) are known, a “pseudo-stress” range is defined that makes use of an effective strain range with an elastic Poisson’s ratio multiplied by the linear modulus to obtain $\Delta\sigma_{\text{eff}}$.

The second parameter considered here is the Findley model [5], a “critical-plane” type of parameter that is defined in terms of the maximum shear stress amplitude on a given plane and the maximum normal stress on that plane multiplied by an adjustable factor, k . It is assumed here that k is a constant over the life range considered.

$$\text{Damage} = \frac{\Delta\tau}{2} + k \sigma_n^{\text{max}} \quad (2)$$

In the implementation of the Findley model, there is some ambiguity in the specific definition of the maximum normal stress term. The early works of Findley, e.g. Ref. 5, all made use of biaxial (bending/torsion) fatigue data that were proportional and in-phase; that is, the normal and shear stress reversals always occurred at the same point in the cycle. From a mechanistic viewpoint, this loading might be expected to produce the most severe fatigue damage, as the microscopic fatigue crack would be subjected to the maximum shear and normal stress simultaneously. However, many components are routinely subjected to multiaxial loadings that are non-proportional or out-of-phase, in which the peak stress components occur at different points in the cycle. It is unclear from Findley's works as to how the maximum normal stress in the cycle should be defined, as (a) the maximum value in the cycle, or (b) the maximum value at the shear reversal points. These two definitions are illustrated conceptually in Figure 2. Using the first definition (designated here as version 1), $\sigma_{\max} = \sigma_C$; with the second definition (version 2), $\sigma_{\max} = \sigma_B$. Note that, if the loading is in-phase (as in Findley's studies), the two definitions are identical. In the present study, both definitions were considered.

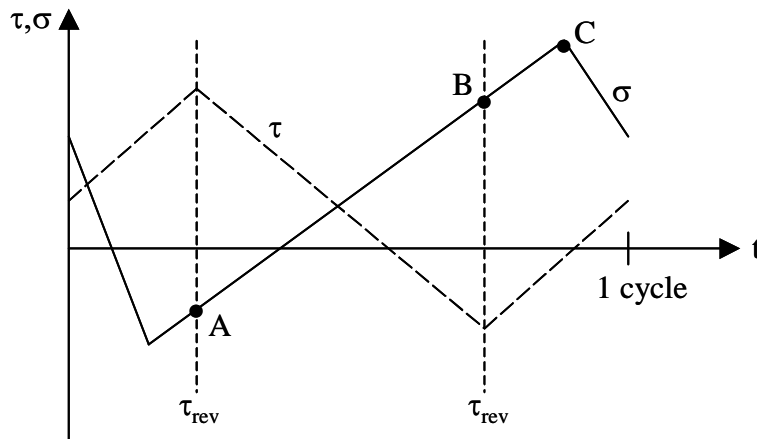


Figure 2. Example biaxial stress history over 1 cycle, showing different methods used to define the normal stress term in the Findley model. The dashed line is the shear stress and the solid line is the normal stress.

The correlations of the three models (MMM model and two versions of the Findley model) with the constant-amplitude data are shown in Figure 3. These plots show the calculated value of the damage parameter (from Eqs. 1 and 2) versus experimental fatigue life for the biaxial data. The curves shown in the plots represent the best fit of the uniaxial data ($R = -1, 0.1, 0.5$), and were used for all damage/life predictions. A successful model can be considered one that “collapses” the biaxial data to the uniaxial curve, with minimal scatter about the curve.

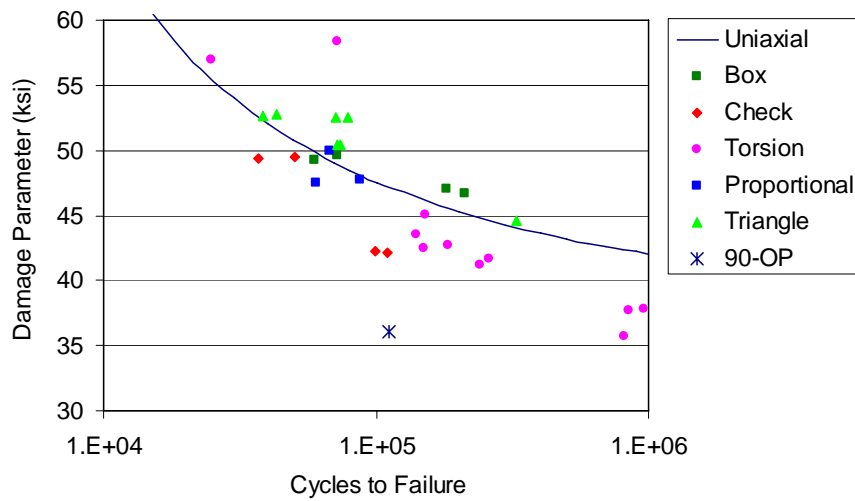
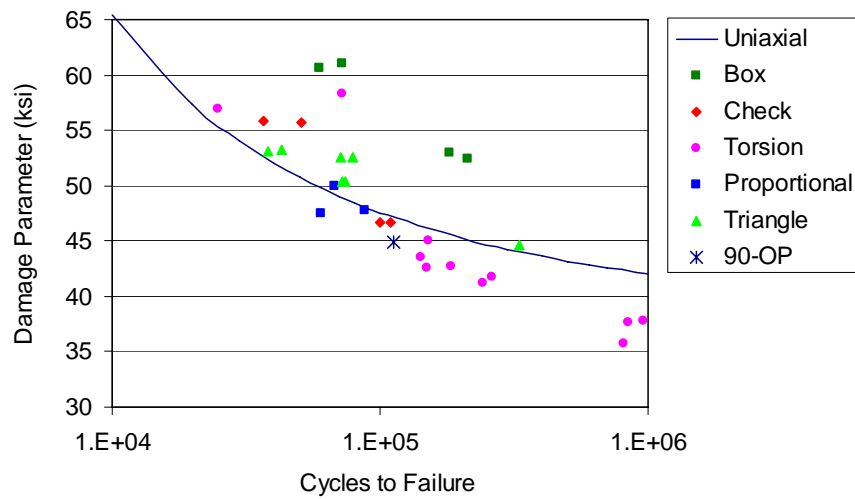
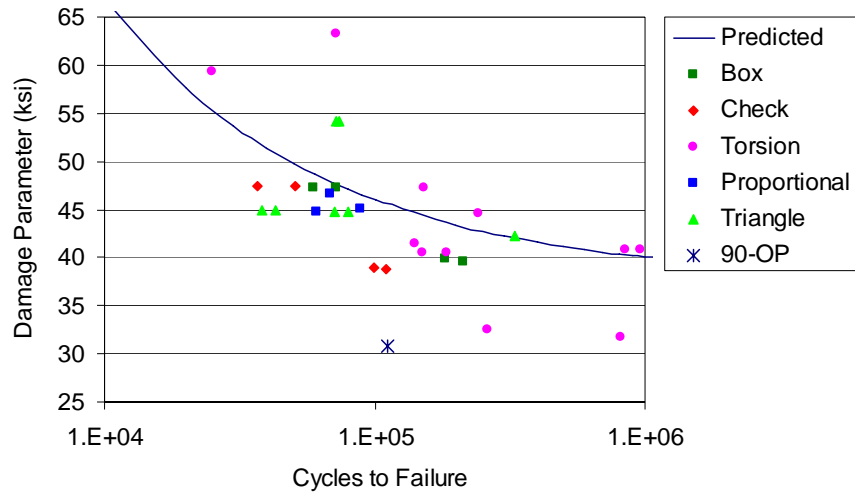


Figure 3. Model correlations for biaxial Ti-6Al-4V data: Modified Manson-McKnight model (top), Findley model – Version 1 (middle), and Findley model – Version 2 (lower).

With reference to Figure 3, it can be seen that the MMM model had difficulty in collapsing some of the torsion data, and was noticeably non-conservative in the damage predictions for the check path and in particular the circle (90-OP) path. The two Findley models produced slightly better overall correlation of the biaxial data, but there are some clear differences between the two versions for some of the non-proportional load paths. In particular, version 1 was highly conservative in its damage predictions for the box path. Version 2 showed much better agreement for the box path; however, the results for the check and circle (90-OP) paths were somewhat non-conservative. The remaining load paths were insensitive to the normal stress definition.

The results shown in Figure 3 indicate that the Findley critical plane parameter can be used effectively for the purposes of multiaxial fatigue damage/life prediction; however, the appropriate definition of the maximum normal stress term appears to be dependent on the particular load path. If this issue can be resolved (in essence, allowing the normal stress definition to vary based on the characteristics of the load path), a very accurate model will result.

To gain a better understanding of the effect of load path characteristics on the normal stress definition in the Findley model, the stress histories on the critical plane for the box, circle, and check paths are shown in Figures 4 – 6 for the two versions of this model. Note that for the box path (Fig. 4), the critical plane identified by version 2 was a plane on which the two stress components were essentially in phase (the sign of the shear stress is arbitrary), while in version 1, σ_{\max} occurred on the critical plane when $\tau = 0$. Recall that version 1 overestimated the damage caused by this path, while version 2 was very accurate. With regard to the circle path (Fig. 5), however, version 1 displayed the same characteristics but provided a better damage estimate than version 2. With version 2, the two stress components were not in phase on the critical plane. In the case of the check path (Fig. 6), version 1 showed slightly better correlation than version 2. Again, the two stress components were not in phase on the critical plane for version 2. This may indicate that a distinction could be drawn by considering the stress histories on the critical plane for version 2 (i.e., checking whether the components are approximately in-phase); however, more evidence would be required to substantiate this claim.

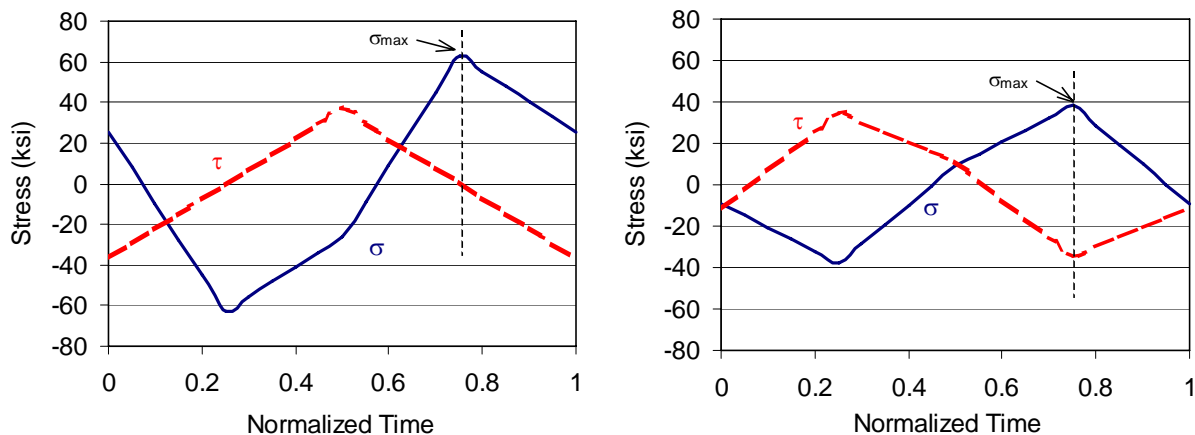


Figure 4. Stress histories on the critical plane for the box path (exp. life = 65,900 cycles). Left: Findley model, Version 1 (plane = 158° , DP = 60.4 ksi, predicted life = 14,400 cycles). Right: Findley model, Version 2 (plane = 122° , DP = 49.3 ksi, predicted life = 63,600 cycles).

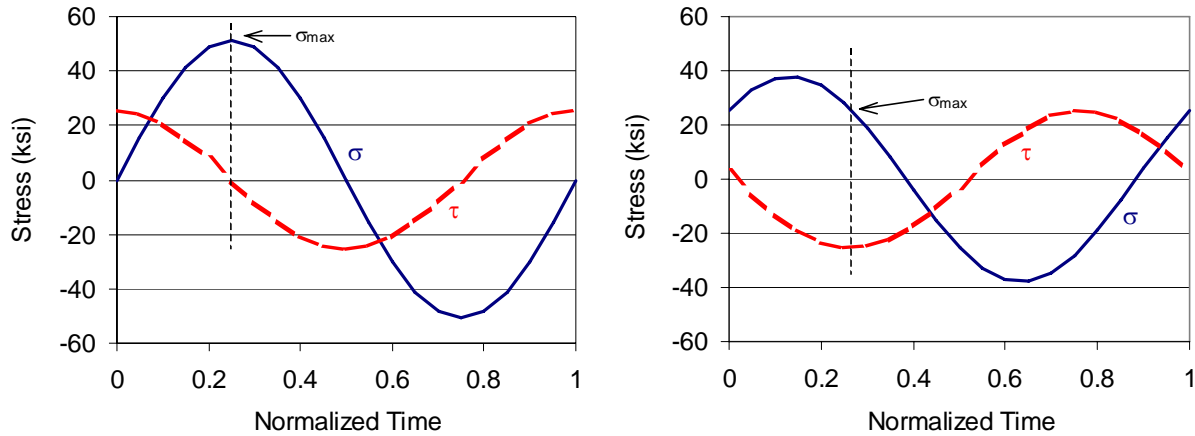


Figure 5. Stress histories on the critical plane for the circle path (exp. life = 111,783 cycles). Left: Findley model, Version 1 (plane = 0°, DP = 44.9 ksi, predicted life = 229,005 cycles). Right: Findley model, Version 2 (plane = 42°, DP = 36.1 ksi, predicted life = 8.55×10^8 cycles).

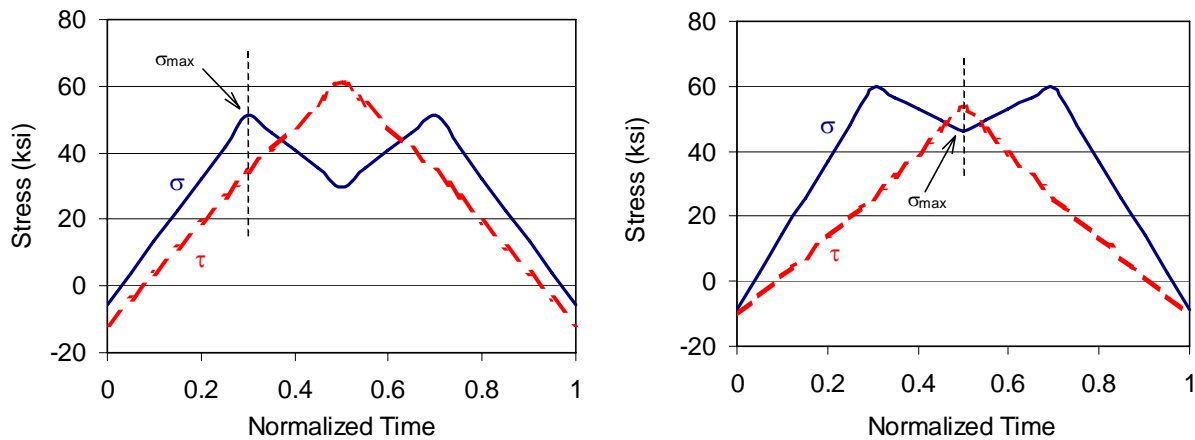


Figure 6. Stress histories on the critical plane for the check path (exp. life = 43,700 cycles). Left: Findley model, Version 1 (plane = 6°, DP = 55.7 ksi, predicted life = 24,100 cycles). Right: Findley model, Version 2 (plane = 12°, DP = 49.4 ksi, predicted life = 64,100 cycles).

SIMULATED MISSION TEST RESULTS AND MODEL COMPARISONS

The simulated “mission” histories were constructed based on two of the non-proportional load paths considered in the previous analysis: the box path and check path. These mission histories were designed to be representative of actual service histories, in which numerous low-damage (HCF) cycles are often coupled with a relatively few highly-damaging (LCF) cycles. The intent of this portion of the study was to experimentally assess the interaction effect between LCF and HCF cycles within a multiaxial stress state, and to provide experimental evidence for evaluating the effectiveness of the multiaxial models under more realistic loading conditions.

The mission histories considered here consisted of one prominent LCF cycle (box or check path) and several small HCF cycles (subcycles), as shown in Figure 7. The differences between the HCF and LCF cycles incorporated both cycle “shape” (or path) and amplitude of loading. The cycle shapes and stress levels were selected to generate different critical planes between the HCF and LCF cycles (as predicted by the Findley models) and to provide a clear distinction in the amount of fatigue damage generated by each cycle. Here, the stress levels for the LCF cycles coincided with the levels applied in the previous analysis, resulting in LCF lives on the order of $10^4 - 10^5$ cycles. The stress levels for the HCF cycles were defined based on Findley model predictions to result in fatigue lives on the order of $10^8 - 10^9$ cycles.

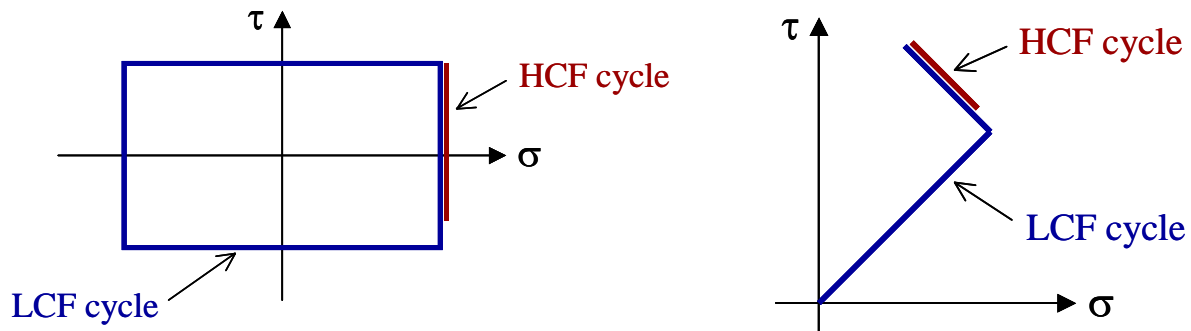


Figure 7. Simulated mission histories, showing LCF and HCF cycles for the box path and the check path.

Three sets of mission history tests were conducted, two based on the box path and one based on the check path. Box mission 1 consisted of 1 LCF cycle and 50 HCF cycles, while box mission 2 consisted of 1 LCF cycle and 5 HCF cycles (the same HCF cycle was used in the two missions). The check mission consisted of 1 LCF cycle and 50 HCF cycles. The mission histories were applied repeatedly to solid round bars of Ti-6Al-4V until complete specimen failure (fracture). The mission life is defined as the number of “missions” completed prior to fracture, or equivalently the number of applied LCF cycles. Two tests were conducted for each mission.

The experimental results for the mission tests are shown in Table 1, along with the predicted mission lives using the three models described previously. In calculating the mission lives, a linear damage rule was used to sum the damage from the cycles in the history. Also shown in the table are the predicted “critical planes” from the Findley models for the LCF and HCF cycles. Note that the critical planes do not coincide. In general, when using a critical plane model, the damage must be summed on each plane for the entire history to determine the mission critical plane. In all cases considered here, the mission critical planes coincided with the LCF critical planes for the Findley models and the predicted mission lives for all models were approximately equal to the predicted LCF cycle lives. This results from the fact that all HCF cycles were predicted to cause very little damage in comparison to the LCF cycles. Even when 50 HCF cycles are applied to every 1 LCF cycle, the increase in predicted fatigue damage is negligible.

Although all the models predicted very little influence from the HCF cycles, the experimental results for the box missions showed the opposite; that is, a significant reduction in mission lives, relative to the LCF lives, were obtained when the HCF cycles were included.

Specifically, when 50 HCF cycles were applied to the box path, the average mission life was approximately 1/3 that of the original LCF life. Even when only 5 HCF cycles were applied, the average mission life was roughly 2/3 of the original LCF life. This indicates that the interaction between the HCF and LCF cycles is much more severe than what is predicted by the linear damage assumption. However, the degree of this HCF/LCF interaction appears to be dependent on the load path. The average mission life for the check mission was approximately equal to the average LCF life when 50 HCF cycles were applied, which is consistent with the linear damage assumption. This would indicate that the load path not only influences the amount of damage caused by the load cycle, but also the rate of damage accumulation under multiaxial states of stress. Clearly, this would add significant complexities to the process of fatigue damage/life estimation.

Table 1
Mission History Results and Model Comparisons

Model	Cycle/Mission	Mission Histories		
		Box 1 1 LCF/50 HCF	Box 2 1 LCF/5 HCF	Check 1 LCF/50 HCF
Mod. Manson McKnight	LCF Life	80,400	81,200	75,400
	HCF Life	∞	∞	∞
	Mission Life	80,400	81,200	75,400
Findley, Version 1	LCF Life/Plane	14,800 (158°)	14,800 (158°)	24,100 (6°)
	HCF Life/Plane	3.0×10^8 (10°)	2.3×10^8 (10°)	1.9×10^8 (48°)
	Miss. Life/Plane	14,800 (158°)	14,800 (158°)	24,100 (6°)
Findley, Version 2	LCF Life/Plane	66,900 (122°)	65,500 (122°)	66,900 (12°)
	HCF Life/Plane	3.0×10^8 (10°)	2.3×10^8 (10°)	1.9×10^8 (48°)
	Miss. Life/Plane	66,900 (122°)	65,400 (122°)	66,600 (12°)
Experimental (2 tests each)	LCF Life	59,432 / 72,360	59,432 / 72,360	50,568 / 36,920
	LCF Average	65,900	65,900	43,700
	Mission Life	19,420 / 21,422	48,787 / 39,480	44,544 / 49,776
	Mission Average	20,400	44,100	47,200

The influence of load path on the interaction effect between HCF and LCF cycles was not anticipated to be this severe. To gain a better understanding of this influence, the relationships between the LCF and HCF cycles for the Findley models are illustrated in Figures 8 and 9 for the box and check missions. These figures show plots of the calculated damage parameter vs. plane orientation (0 to 180°) from the two Findley models. The critical planes for each cycle correspond to the peaks in the curves.

The predicted dominance of the LCF cycle is evident in Figs. 8 and 9. In version 1 of the Findley model, the damage parameter for the LCF cycle has a higher value than that for the HCF cycle on all planes. In version 2, there are a few planes on which the HCF damage parameter is actually higher than the LCF value, but the combined effect of the damage on those planes, when assuming linear damage, is still much less than the damage on the LCF critical plane.

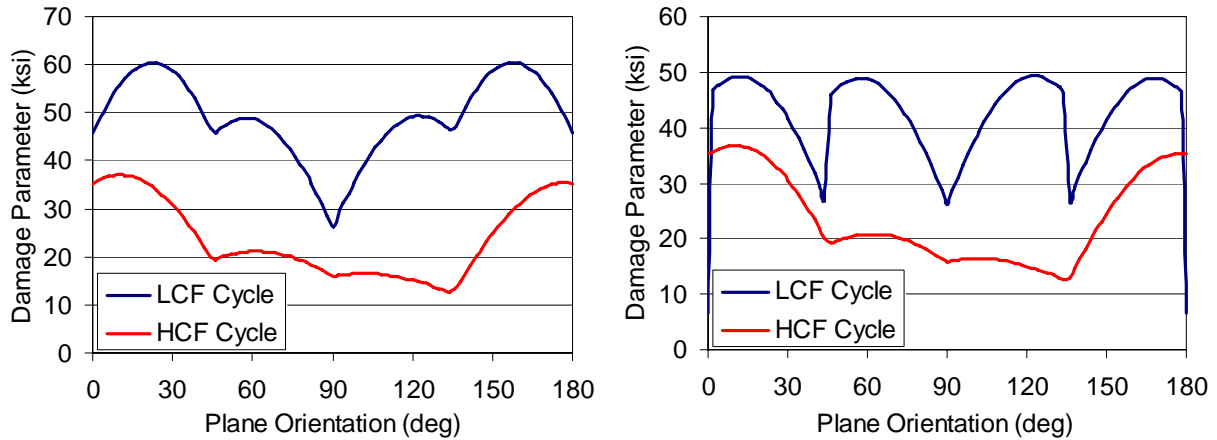


Figure 8. Damage parameter vs. plane orientation for the box mission using the Findley models: version 1 (left) and version 2 (right).

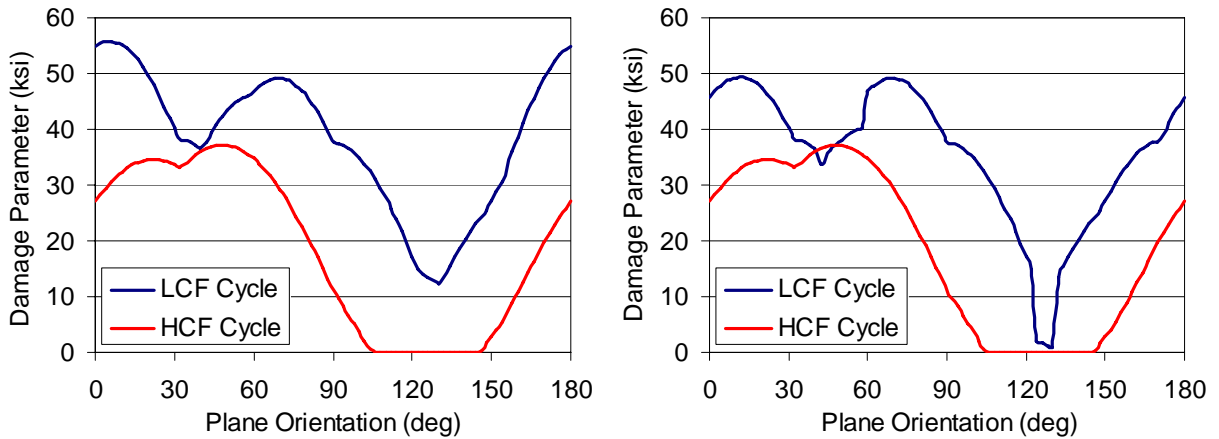


Figure 9. Damage parameter vs. plane orientation for the check mission using the Findley models: version 1 (left) and version 2 (right).

In comparing the two versions of the Findley model, it is of interest to note that the number of critical regions calculated by version 2 is double that of version 1. Considering the box mission, the LCF damage is symmetric about 90° for both models. Version 1 thus predicts two critical regions (two approximately equal peaks in the curve). Version 2, however, predicts four critical regions. While the damage for the check mission is not symmetric, version 2 still predicts two critical regions to one predicted by version 1. This is an important characteristic when viewing the process of fatigue crack initiation from a stochastic viewpoint. The more critical “planes” or “regions,” the more likely (statistically) a fatigue crack would be to initiate at a given load level in a given number of cycles. These characteristics of the two versions of the Findley model may also provide some input into the appropriate definition of the normal stress term for a given load path.

SUMMARY AND CONCLUSIONS

Both the modified Manson-McKnight and Findley models provided adequate estimations of fatigue damage under constant-amplitude, biaxial stress states. The Findley model, in particular, can provide very good estimations provided the correct definition of maximum normal stress is used, which is dependent on the load path. However, more research is required to provide a reliable method for characterizing the specific dependence.

In contrast to their success with constant amplitude loadings, neither model accurately accounted for the HCF/LCF interactions observed experimentally in the box mission histories. However, this is more likely a problem with the damage accumulation rule than the models themselves. The load path was also found to influence the degree of interaction, as the check mission produced very little interaction between LCF and HCF cycles. This is a very important point, as it suggests that damage accumulates nonlinearly for some types of loading.

Although not an issue for the mission histories considered in this study, the method of cycle counting is another important consideration when using critical plane models with variable load histories. Most critical plane models include stress-strain terms from more than one tensorial location. The choice of what term in the damage parameter to use as the "cycle-counter" for the extension of an algorithm such as rainflow counting is not always obvious. In this study, the cycles were clearly defined; however, in more complex histories, the choice of counting term can affect the identification of complete cycles, and consequently the predicted damage/life.

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