Rules for Expected Mean Squares

1. F-statistics
   $F_A = \frac{MS_N}{MS_D} \sim F(df_N, df_D)$ where MS_N and MS_D are chosen so that $E[MS_N] - E[MS_D]$ is a *sole* function of the variance component of A, say ϕ_A or σ_A^2.

2. Unrestricted and Restricted Mixed Models

   (a) Restricted Mixed Model
   Assumes, for example, $\sum_{i=1}^{a}(AB)_{ij} = 0$ where A is fixed and B is random.

   (b) Unrestricted Mixed Model
   Assumes, for example, $(AB)_{ij}$ are iid $N\left(0, \sigma_{AB}^2\right)$ random variables.

   (c) $E[MS]$ depends on what type of mixed model is being used!

Prerequisites of the $E[MS]$ Algorithm

1. Assumptions

   (a) Restricted Mixed Model

   (b) Balanced Design

2. Prerequisites

   (a) Notation for terms in the model.
   main effects - A_i, B_j, . . .
   interaction effects - AB_{ij}, ABC_{ijk}, . . .
   nested effects - $A(B)_{i(j)}$, $A(BC)_{i(jk)}$, . . .
   random *error* - $\epsilon_{r(ijk\ldots)}$

   (b) For each term in the model divide the corresponding subscript(s) into one of three classes.

      i. Live (L) subscripts appear in the term but are not in parentheses.

ii. Dead (D) subscripts appear in the term and are in parentheses.

iii. Absent (A) subscripts appear in the model but are not in the term.

(c) Degrees of freedom for each term.

$$df_A = \left(\prod_{i=1}^{d} D_i \right) \left(\prod_{j=1}^{l} (L_j - 1) \right)$$

where $D_1, \ldots, D_d$ represent the number of levels associated with each dead subscript in A and $L_1, \ldots, L_l$ are analogously defined for each live subscript in A.

(d) Fixed (F) or random (R) effects?

i. *fixed* effects - ϕ_A, ϕ_{AB}, . . .

ii. *random* effects - σ^2_A, σ^2_{AB}, . . .

iii. Terms containing random effects are random.

iv. If one or more factors in a term appears in parentheses (e.g. nested effects) then there can be no interaction between those factors appearing in the term (e.g. there is no AB interaction if $A(B)$ is in the model).

Determining $E[MS]$ for Each Term

1. Construct the following table.

2. For each row (term) put a 1 under dead subscripts appearing in the term.

3. For each row (term) put a 0 under fixed factor subscripts appearing in the term.

4. For each row (term) put a 1 under random factor subscripts appearing in the term.

5. In the remaining (blank) cells write the number of levels corresponding to the column.

6. Determine $E[MS]$ for each term.

   (a) Cover column(s) corresponding to *all* live subscripts appearing in the term.

   (b) For each row that contains *at least the same* subscripts as those in the term being considered multiply the visible numbers and then multiply this product by the corresponding variance component.

   (c) The sum of these components is $E[MS]$.

EMS Using SAS and Minitab

1. A two-factor factorial with one factor random.

   (a) SAS Analysis

```
proc glm data=raw;
  class B A;
  model response=A B A*B;
  random B A*B;
run;

Source  Type III Expected Mean Square
A       Var(Error) + 4 Var(B*A) + Q(A)
B       Var(Error) + 4 Var(B*A) + 12 Var(B)
B*A     Var(Error) + 4 Var(B*A)
```

   (b) Minitab Analysis

```
MTB > ANOVA c1 = A B A*B;
SUBC>   Random B;
SUBC>   Restrict;
```

```
SUBC>   EMS.

Source  Variance Error Expected Mean Square
         component  term  (restricted model)
 1 A               3    (4) +  4(3) + 12Q[1]
 2 B      0.05511  4    (4) + 12(2)
 3 A*B    1.03484  4    (4) +  4(3)
 4 Error 0.85335        (4)
```

2. A two-stage nested design.

(a) SAS Analysis

```
proc glm data=raw;
  class nest trmt;
  model response=trmt nest(trmt);
  test h=trmt e=nest(trmt);
  random nest(trmt);
run;

Source Type III Expected Mean Square
TRMT Var(Error) + 3Var(NEST(TRMT)) + Q(TRMT)
```

```
NEST(TRMT) Var(Error) + 3Var(NEST(TRMT))
```

(b) Minitab Analysis

```
MTB > ANOVA c1 = T N(T);
SUBC>   Random N;
SUBC>   Restrict;
SUBC>   EMS.

Source Variance Error Expected Mean Square
       component  term  (restricted model)
 1 T               2    (3) + 3(2) + 12Q[1]
 2 N(T)  0.7450    3    (3) + 3(2)
 3 Error 0.8533         (3)
```